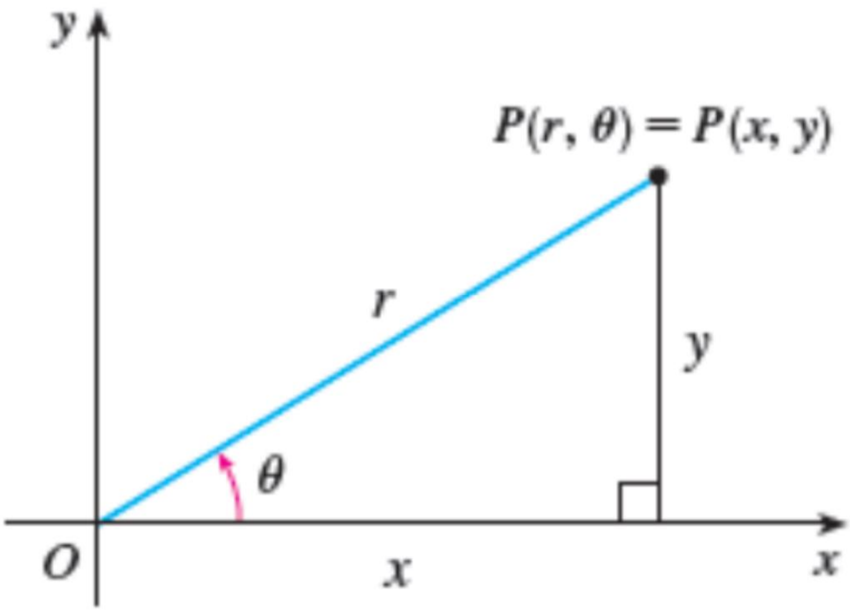
Section 15.7:
Triple Integrals In Cylindrical Coordinates

What We’ll Learn In Section 15.7

1. What are cylindrical coordinates?
2. Triple integrals in cylindrical coordinates

1. What are cylindrical coordinates?

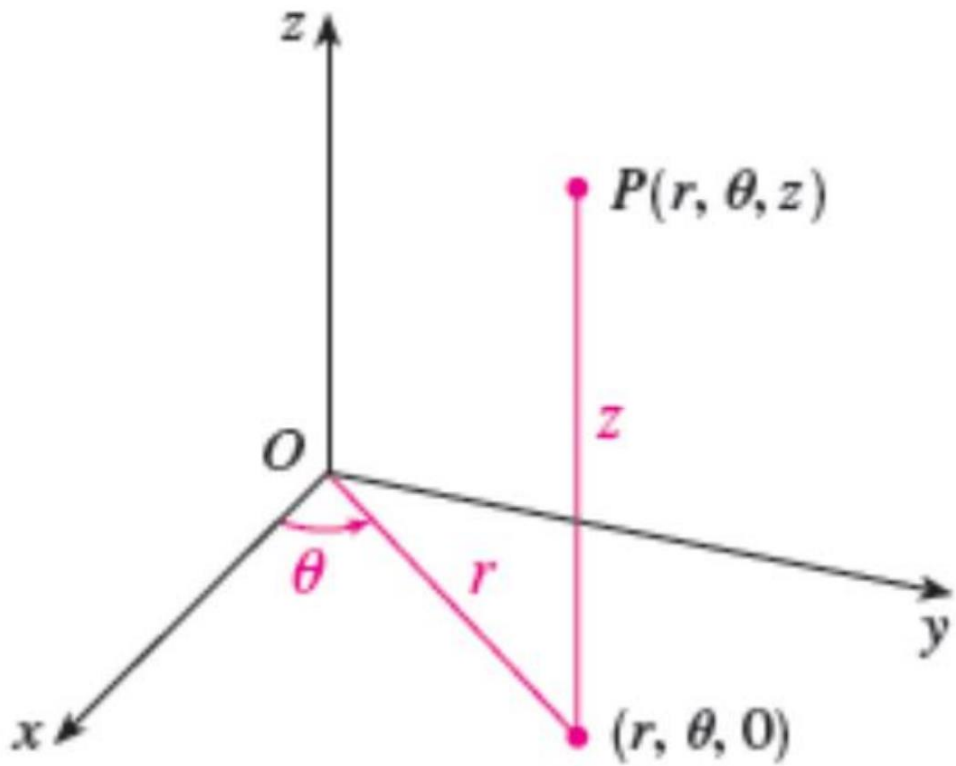


$$x = r\cos\theta \qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

1. What are cylindrical coordinates?

The cylindrical coordinates of a point



$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

1. What are cylindrical coordinates?

To convert from cylindrical to rectangular coordinates

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

to convert from rectangular to cylindrical coordinates

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

1. What are cylindrical coordinates?

Ex 1:

a) Plot the point with cylindrical coordinates $(2, \frac{2\pi}{3}, 1)$ and find its rectangular coordinates.

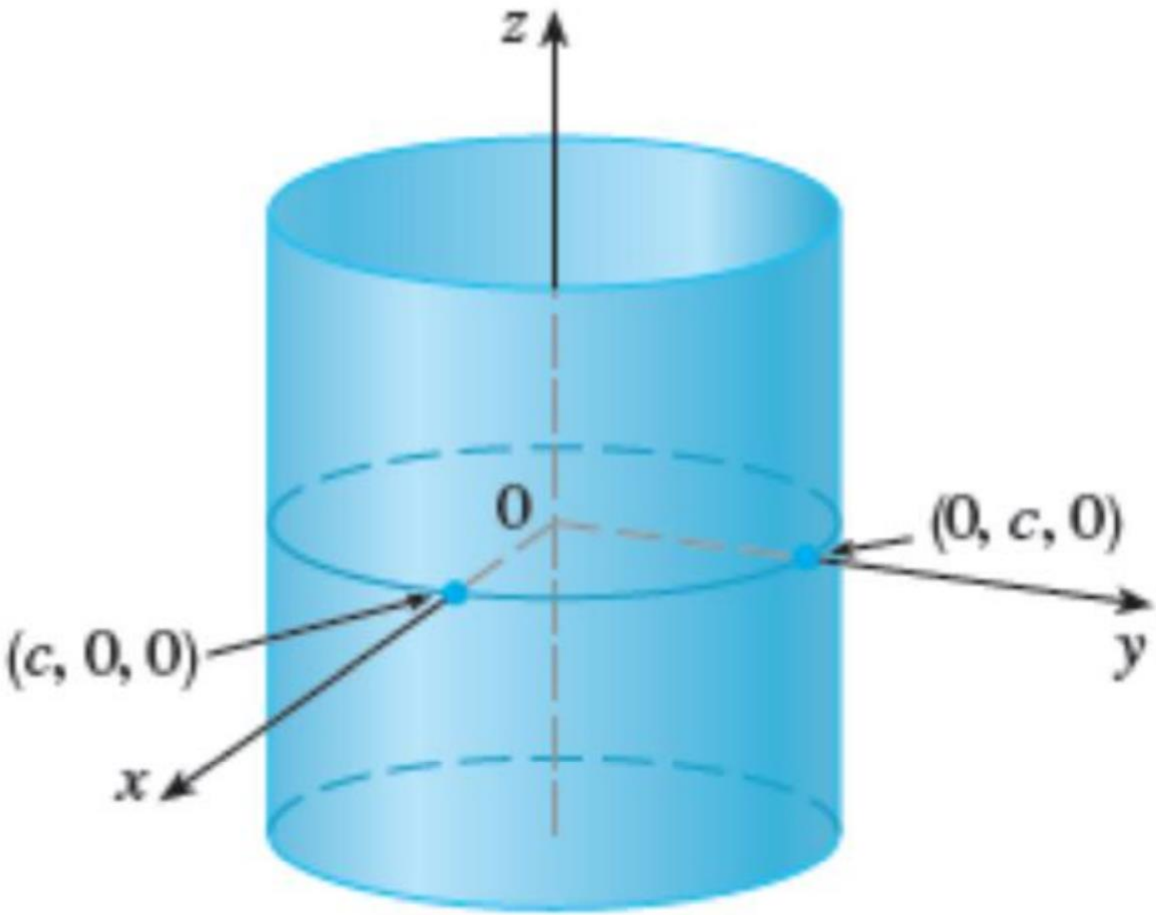
1. What are cylindrical coordinates?

Ex 1:

b) Find cylindrical coordinates of the point with rectangular coordinates $(3, -3, -7)$.

1. What are cylindrical coordinates?

$r = c$, a cylinder

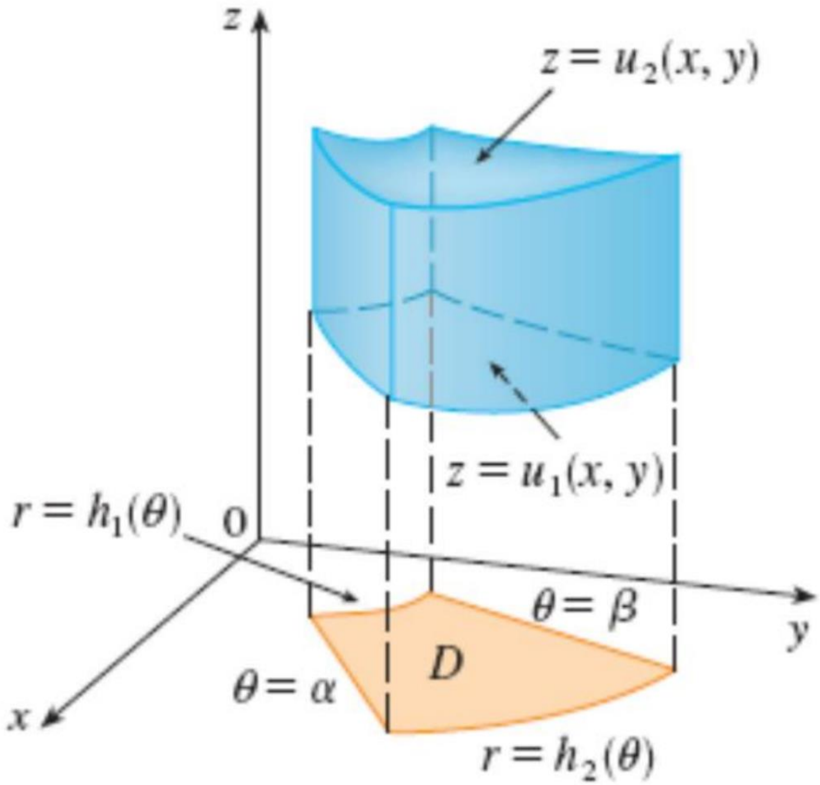


1. What are cylindrical coordinates?

Ex 2: Describe the surface whose cylindrical coordinates satisfy the equation $z = r$.

2. Triple integrals in cylindrical coordinates

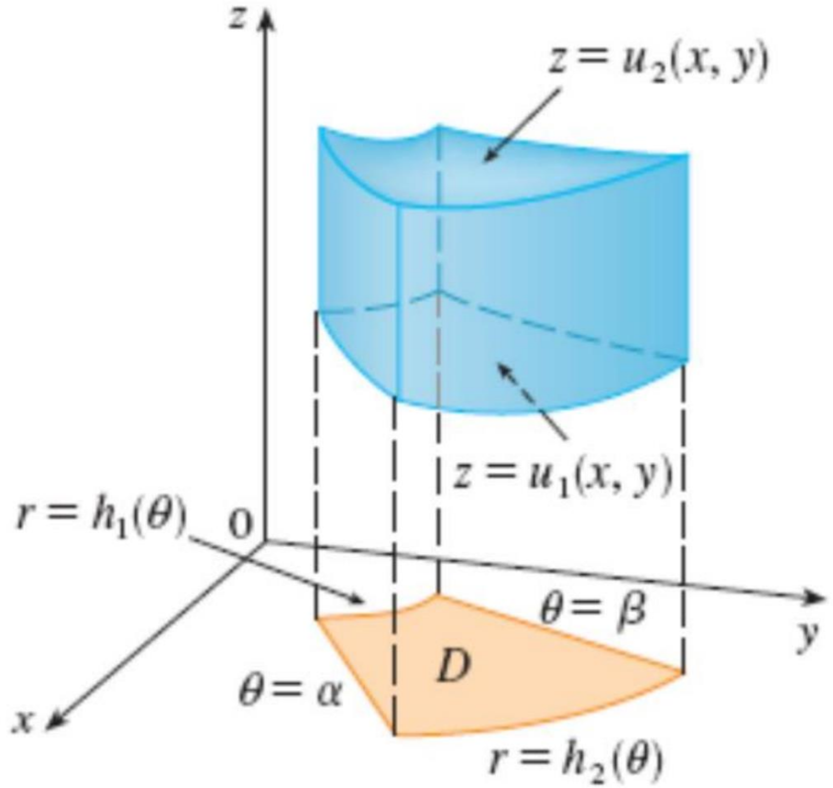
Suppose that E is a type 1 region whose projection D onto the xy-plane is conveniently described in polar coordinates



$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,\, y)}^{u_2(x,\, y)} f\,(x, y, z)\, dz\right] dA$$

2. Triple integrals in cylindrical coordinates

Suppose that E is a type 1 region whose projection D onto the xy-plane is conveniently described in polar coordinates

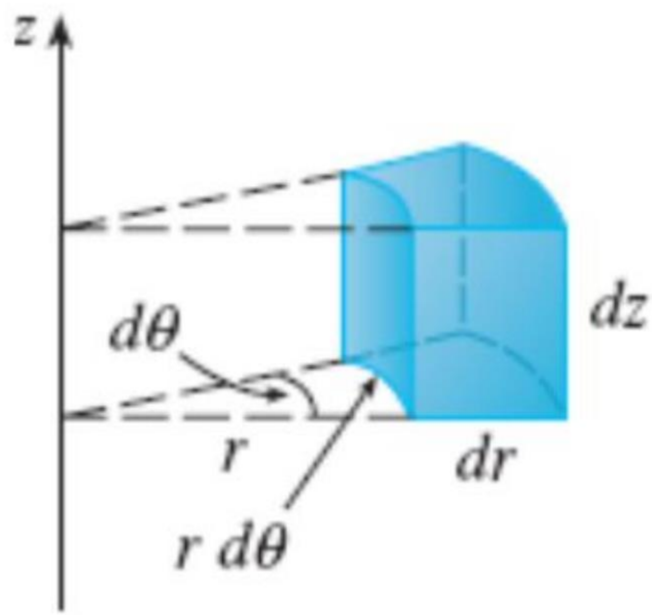


$$\iiint_E f(x, y, z)\ dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta,\, r\sin\theta)}^{u_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\ r\, dz\, dr\, d\theta$$

2. Triple integrals in cylindrical coordinates

Volume element in cylindrical coordinates:

$$dV = r\,dz\,dr\,d\theta$$



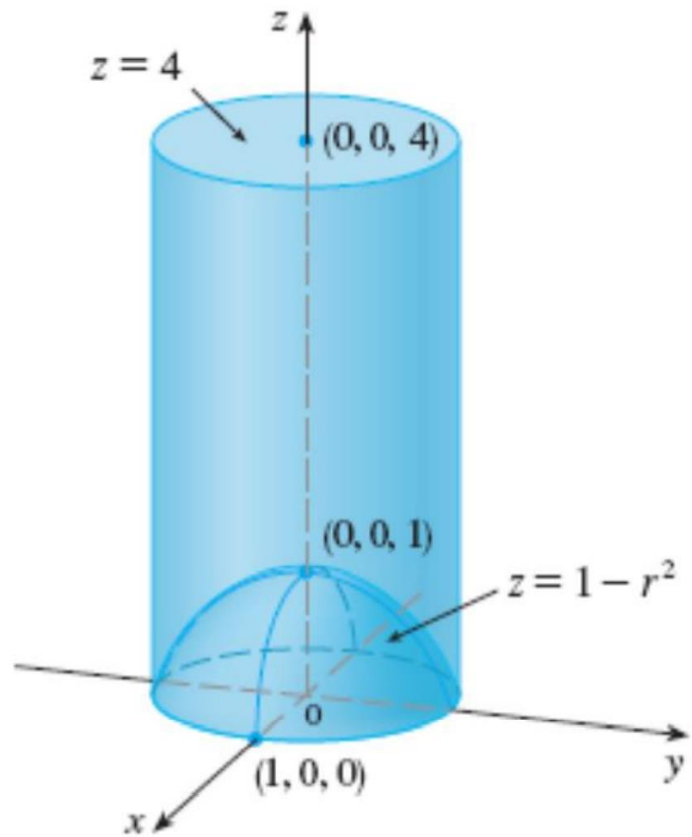
$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

2. Triple integrals in cylindrical coordinates

Ex 3: A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. Its density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.



2. Triple integrals in cylindrical coordinates

Ex 4: Evaluate $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}\left(x^2+y^2\right)dzdydx$